

# Effect of Small Branch Inclination on Gas–Liquid Flow Separation in T Junctions

Marcel Ottens, Huub C. J. Hoefsloot, and Peter J. Hamersma

Dept. of Chemical Engineering, University of Amsterdam, 1018 WV Amsterdam, The Netherlands

*A model developed calculates the mass intake fraction of gas and liquid phase during gas–liquid flow with small liquid holdup values ( $\epsilon_L \leq 0.1$ ) through regular T junctions with small branch inclinations. It was derived from the steady-state macroscopic mechanical energy balance (extended Bernoulli equation) applied to the “inlet-to-run” streamline and “inlet-to-branch” streamline of both gas and liquid phases. The model results are compared with experimental data of the system air/water–glycerol (0, 33, and 60 wt. %) flowing through a regular ( $D_1 = D_2 = D_3 = 0.051$  m) T junction with branch inclinations ranging from  $0^\circ$  to  $0.5^\circ$ . It was found that the gas–liquid flow split behavior is affected strongly by the liquid viscosity and by branch inclinations of  $0.1^\circ$  and higher. The results predicted with the model agree well with experimental results obtained in a regular T junction with a horizontal inlet run and an inclined side arm.*

## Introduction

Gas–liquid (G/L) flow in T junctions is relevant to various industries, such as the petrochemical industry, the chemical process industry, and the nuclear energy industry. Gas–liquid flow in T junctions may cause unexpected phenomena. Generally, one expects that the liquid phase will split in the same ratio over the side arm (branch) and main pipe (run) as the gas phase. If, for example, the gas flow is split equally over the run and branch, it might occur that for a G/L flow with a low liquid loading *all* liquid enters the branch. However, at a slightly different gas-flow split ratio *all* liquid enters the run, that is, the so-called *flip-flop* effect. This effect was reported first by Oranje (1973), who investigated the *route preference* of liquid condensate during the transport of natural gas in natural-gas transportation pipeline networks. The existence of liquid condensate in natural gas resulted in separation problems. All delivery stations were equipped with G/L separators, but only a few collected liquid that could be attributed to this unequal phase splitting in T junctions. Similar route preference problems may occur during enhanced oil recovery (EOR), where steam injection is applied to lower the viscosity of the residual oil left in the reservoir. Another field of interest and research effort is found in the nuclear energy

industry. The so-called loss-of-coolant accidents (LOCAs) can cause the appearance of a two-phase G/L mixture in the cooling bundles of the reactor. Preferential flow of all the gas in one direction after a T junction may lead to insufficient cooling capacity and consequently to a runaway. The unequal phase splitting is especially pronounced for small liquid holdups. Although in many industrial applications T junctions with a slightly inclined branched pipe occur, for example,  $0$ – $2^\circ$ , publications considering this inclination effect can scarcely be found in the open literature. Only recent articles by Panmarcha et al. (1996) and Marti and Shoham (1997) deal with the subject of small branch inclination angles. However, it is well known that small inclination angles have large effects on the flow regime, and the transition boundaries of G/L flow in straight pipes (Grolman, 1994). In this article we pay special attention to experiments and a model dealing with the effect of small branch inclination angles.

The junction geometry is an important variable. Different shapes can be characterized through orientation of the branched pipe—vertical, horizontal, or inclined from the horizontal under an angle  $\alpha$ ; angle between the branch and the main pipe  $\varphi$ ; the rounding ratio of the edge between the main arm and the branch  $r$ . The flow regime at the inlet of the junction is also an important parameter that influences the flow split. The large variety of involved physical quantities has contributed to the fact that no general applicable

Correspondence concerning this article should be addressed to P. J. Hamersma.  
Current address of M. Ottens: Kluiver Laboratory for Biotechnology, Delft University of Technology, Julianalaan 67, 2628 BC Delft, The Netherlands.

model is available that predicts the G/L flow split in junctions under all operating conditions. Nevertheless, individual research groups have made progress in restricted areas of two-phase flow in T junctions.

### Significance of previous work

During the last three decades, a large amount of experimental data has become available in the literature, all obtained from experiments in junctions of different shapes and under a wide range of flow conditions. Roberts (1994) summarizes the experimental matrix covered so far. It can be concluded that almost no data are available for regular horizontal T junctions with small inclination angles for the side arm, from which we know that they can have a large impact on the G/L flow, its flow regime, and associated liquid holdup and pressure drop. The systems also are mainly limited to air/water flow with low viscosities of the liquid phase.

Models to predict the G/L flow split in T junctions are given by, for example, Saba and Lahey (1984) and Seeger et al. (1986); Azzopardi and Whalley (1982); Azzopardi et al. (1987); Shoham et al. (1987, 1989); Ballyk and Shoukri (1990); Hart et al. (1991); Ottens et al. (1994); and Roberts et al. (1995, 1997).

Saba and Lahey (1984) developed a model restricted to homogeneous flow (e.g., dispersed bubbly flow) that is not applicable to stratified-wavy separated flow with low liquid holdups. Furthermore, their model contains a large set of empirical correlations. Seeger et al. (1986) extended the model of Saba and Lahey (1984) and gave empirical relations for horizontal and 90° downwardly inclined branches. The model of Shoham et al. (1987, 1989) is an example of the so-called geometrical models valid for annular and stratified-wavy flow. The flow split in this model is to a large extent governed by a virtual vertical split line through the inlet, dividing both the gas and liquid between the branch and the run. The value of the distance of the separation line from the wall for the liquid phase and the gas phase is determined by the centrifugal forces of the gas and the liquid phases. This model is not always consistent with experimental findings. Hwang (1988) used the concept of Shoham (1987, 1989) of dividing streamlines and introduced zones of influence to calculate the two-phase flow distribution at a T junction. Ballyk and Shoukri (1990) developed a model for the annular flow regime. However, the pressure distribution over the T junction should be known to predict the splitting ratio, restricting its applicability. Azzopardi and Whalley (1982) studied the annular flow regime in a horizontal T junction with the branch inclined from the main pipe from -90° to 90°. An increase in the mass intake fraction of the liquid phase into the branch  $\lambda_L$  at constant mass intake fraction of the gas phase into the branch  $\lambda_G$  was found due to the difference in liquid-film thickness being present in annular flow. They developed a model based on the observations of McNown (1954) that gas and liquid emerging in the branch originates from a segment of the main tube near the branch. Azzopardi (1989) observed the so-called "film-stop" of the liquid film in the run of the T junction. As a result, an extra amount of liquid enters the branch. Roberts (1995, 1997) improved the model of Azzopardi and Whalley (1982), taking the distribution of the liquid film into account. Hart et al. (1991) developed a

model to predict the mass flow rate entering the branch that showed better results for the range of low values of the liquid holdup. The model is based on the macroscopic mechanical energy balance. Important parameters are the ratio between the kinetic energies per unit of volume of the gas and liquid phase in the inlet  $\kappa$  and the junction energy dissipation factor  $\lambda_0$ . Loss coefficients for the liquid phase are equal to those of the gas phase and are described by single-phase correlations. Ottens et al. (1994) extended the model's range applicability by developing correlations for the liquid-phase loss coefficient.

### Scope of this work

In conclusion we can state that little or no experimental data are available on gas-liquid flow split in regular T junctions with an inclined branch and low liquid loading under conditions of moderate superficial gas velocities. There also isn't much information on the influence of the viscosity of the liquid phase on the flow split behavior. Furthermore, under certain process conditions and positive branch inclination angles, intermittent flow will occur in the branch, influencing the flow regime in the inlet and the run. In this article, we consider only the stratified-wavy flow regime.

None of the models mentioned are able or suitable to predict the maldistribution of the G/L flow accurately in T junctions under low liquid holdup conditions and small branch inclination angles. This brings us to the aim of this article, which is to develop a model describing the G/L flow split in a T junction specifically for low liquid holdups; to correctly model the influence of the inclination of the branch arm on the G/L flow split in a T junction; and to investigate the influence of the liquid-phase dynamic viscosity on the G/L flow split in a T junction. Therefore this article shows both the experimental and theoretical results concerning regular T junctions with small branch inclination angles. An experimental setup is presented to measure the flow split and pressure distribution. Further, the model to be developed will be based on the model of Hart et al. (1991) and a criterion for net positive liquid transport. Finally, a comparison is made between the experimental and model results.

### Gas-Liquid Flow Split Model

Hart et al. (1991) showed that the macroscopic energy balance (extended Bernoulli equation) can be applied to the steady-state continuous separated two-phase flow of a gas and liquid phase in a dividing T junction (see also Figure 1):

$$\frac{1}{2} \left( \frac{\langle v_1^3 \rangle}{\langle v_1 \rangle} - \frac{\langle v_i^3 \rangle}{\langle v_i \rangle} \right)_j + g(z_1 - z_i)_j + \frac{1}{\rho_j} (P_1 - P_i)_j = (\hat{E}_{v1i})_j \quad (1a-d)$$

where  $i$  denotes run (2) or branch (3) and  $j$  the gas (G) or liquid (L) phase. Equation 1 represents the conservation of energy along four different streamlines:

- Inlet-to-run gas streamline (1a)
- Inlet-to-branch gas streamline (1b)
- Inlet-to-run liquid streamline (1c)
- Inlet-to-branch liquid streamline (1d)

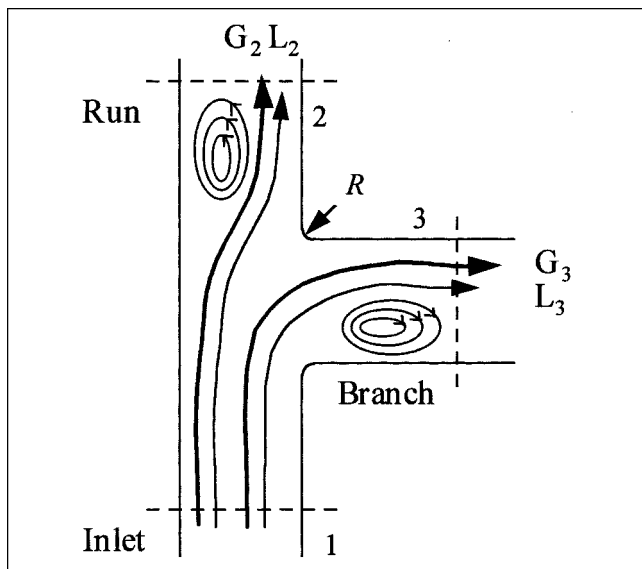


Figure 1. Gas and liquid streamlines occurring during G/L flow through a regular dividing T junction.

Bold lines, gas; thin lines, liquid.

Combining Eqs. 1a–1d by subtracting the equations for the liquid streamlines and subtracting the equations for the gas streamlines under the assumption that:

$$(P_2 - P_3)_G = (P_2 - P_3)_L \quad (2)$$

results in

$$\begin{aligned} & \frac{1}{2} \rho_G \left( \frac{\langle v_3^3 \rangle}{\langle v_3 \rangle} - \frac{\langle v_2^3 \rangle}{\langle v_2 \rangle} \right)_G + \frac{1}{2} \rho_L \left( \frac{\langle v_3^3 \rangle}{\langle v_3 \rangle} - \frac{\langle v_2^3 \rangle}{\langle v_2 \rangle} \right)_L \\ & + \rho_G g(z_3 - z_2)_G + \rho_L g(z_3 - z_2)_L \\ & = \rho_G (\hat{E}_{v12} - \hat{E}_{v13})_G - \rho_L (\hat{E}_{v12} - \hat{E}_{v13})_L \end{aligned} \quad (3)$$

The junction energy loss  $\hat{E}_v$  is defined by (Bird et al., 1960):

$$\hat{E}_{v1i,j} = k_{1i,j} \frac{1}{2} \frac{\langle v_{jl}^3 \rangle}{\langle v_{jl} \rangle}, \quad (4)$$

with  $k$  the friction loss coefficient in a junction of one single phase. We define the velocity profile factor  $\beta$  by

$$\beta = \frac{\langle v^3 \rangle}{\langle v \rangle^3}, \quad (5)$$

and  $\kappa$ , the ratio of the kinetic energies of the gas and the liquid at the inlet per unit of volume,

$$\kappa = \frac{\rho_{G1} \langle v_{G1}^3 \rangle / \langle v_{G1} \rangle}{\rho_{L1} \langle v_{L1}^3 \rangle / \langle v_{L1} \rangle} = \frac{\beta_{G1} \rho_{G1} \langle v_{G1} \rangle^2}{\beta_{L1} \rho_{L1} \langle v_{L1} \rangle^2}. \quad (6)$$

The velocity profile factor  $\beta$  strongly depends on the distribution of the velocities in the cross section of the phase involved and may have values between 1.0 in the case of turbulent flow and 1.5–2.3 in the case of laminar flow of a thin liquid film in a pipe (see, e.g., Grolman, 1994). The mass intake fractions into the branch of the gas and the liquid phase are defined by

$$\lambda_G = \frac{\Phi_{MG3}}{\Phi_{MG1}} = \frac{\langle v_{G3} \rangle \epsilon_{G3}}{\langle v_{G1} \rangle \epsilon_{G1}} \quad (7)$$

$$\lambda_L = \frac{\Phi_{ML3}}{\Phi_{ML1}} = \frac{\langle v_{L3} \rangle \epsilon_{L3}}{\langle v_{L1} \rangle \epsilon_{L1}}. \quad (8)$$

Substituting Eqs. 4–8 into Eq. 3 and taking into account the mass balances of both phases, that is,  $\Phi_{M1} = \Phi_{M2} + \Phi_{M3}$ ,  $\Phi_{M3} = \lambda \Phi_{M1}$  and  $\Phi_{M2} = (1 - \lambda) \Phi_{M1}$ , leads to the governing equation for G/L flow split in a regular T junction:

$$\begin{aligned} & \kappa \left\{ \frac{\beta_{G2} \epsilon_{G1}^2}{\beta_{G1} \epsilon_{G2}^2} (1 - \lambda_G)^2 - \frac{\beta_{G3} \epsilon_{G1}^2}{\beta_{G1} \epsilon_{G3}^2} \lambda_G^2 \right\} \\ & - \left\{ \frac{\beta_{L2} \epsilon_{L1}^2}{\beta_{L1} \epsilon_{L2}^2} (1 - \lambda_L)^2 - \frac{\beta_{L3} \epsilon_{L1}^2}{\beta_{L1} \epsilon_{L3}^2} \lambda_L^2 \right\} + \frac{2g}{\beta_{L1} \langle v_{L1} \rangle^2} \\ & \times \left\{ \frac{\rho_G}{\rho_L} (z_{G2} - z_{G3}) - (z_{L2} - z_{L3}) \right\} \\ & = \kappa (k_{13} - k_{12})_G - (k_{13} - k_{12})_L. \end{aligned} \quad (9)$$

For G/L flow with a small liquid holdup ( $\epsilon_L < 0.06$ ) Hart et al. (1991) simplified Eq. 9 by assuming that

- The ratio of the  $\beta$  values of each phase is approximately unity;
- The liquid holdup values in the inlet, run, and branch in the vicinity of the T junction are approximately equal (as a consequence the differences in the gas and liquid heights in the vicinity of the tee are approximately zero);
- The differences of the friction loss coefficients between the run and the branch for the liquid and the gas phase are the same, that is,  $(k_{13} - k_{12})_G = (k_{13} - k_{12})_L$ .

Applying these assumptions to Eq. 9 results, and introduction of an energy dissipation factor  $\lambda_0$  defined by

$$\lambda_0 = \frac{1}{2} (1 + k_{12} - k_{13}) \quad (10)$$

led to the so-called *double-stream model* (DSM):

$$\lambda_L = \lambda_0 + \kappa (\lambda_G - \lambda_0). \quad (11)$$

The values of the friction loss coefficients for a regular T junction were calculated with the correlations of Gardel (1957), which describe the single-phase friction loss coefficients and are generally dependent on the geometry ( $r$ ,  $\varphi$ ) of the junction and the mass intake fraction:

$$k_{12} = 0.03(1 - \lambda_G)^2 + 0.35\lambda_G^2 - 0.2\lambda_G(1 - \lambda_G) \quad (12a)$$

$$k_{13} = 0.95(1 - \lambda_G)^2 + \lambda_G^2 \left( \left\{ 1.3 \cot \left( \frac{1}{2} \varphi \right) \right\} \left\{ 1 - 0.9\sqrt{r} \right\} \right) + 0.8 \lambda_G (1 - \lambda_G) \cot \left( \frac{1}{2} \varphi \right). \quad (12b)$$

In this article, we investigated regular T junctions with a slightly inclined branch. We found that for G/L flow in a regular T junction with a slightly *inclined* branch the ratio  $\beta_{L3}/\beta_{L1}$  has a pronounced effect on the flow split of the liquid phase, while the ratio  $\beta_{L2}/\beta_{L1}$  does not affect the flow split. Although most ratios of the involved  $\beta$  values are approximately unity for T junctions with a slightly inclined branch, the ratio  $\beta_{L3}/\beta_{L1}$  has to be taken into account. This can be understood by the fact that in the case of an inclined pipe a liquid film may have an average velocity larger than zero, but without having a “normal” parabolic velocity distribution as observed during horizontal flow (see also Eq. 14).

With  $\beta_{L3}/\beta_{L1}$  not *a priori* equal to unity and the just mentioned assumptions, Eq. 9 results in an equation similar to Eq. 11, though, with the nonlinear term  $1/2(1 - \beta_{L3}/\beta_{L1})\lambda_L^2$ :

$$\lambda_L = \lambda_0 + \kappa(\lambda_G - \lambda_0) + \frac{1}{2}(1 - \beta_{L3}/\beta_{L1})\lambda_L^2. \quad (13)$$

In Figure 2 we have plotted the values of  $\lambda_L$  vs.  $\lambda_G$  calculated by Eq. 11 and Eq. 13 for three different  $\kappa$  values (0.1, 1, 10), an average value of  $\bar{\lambda}_0 = \int_0^1 \lambda_0 d\lambda_G = 0.07$ , and two typical values of the ratio  $\beta_{L3}/\beta_{L1}$  (1.0 and 2.0). It should be noted that  $\beta_L = 1.0$  for fully developed turbulent flow and  $\beta_L \approx 2.0$  for a sheared laminar liquid film. As shown in Figure 2, the ratio  $\beta_{L3}/\beta_{L1}$  clearly affects the  $\lambda_L - \lambda_G$  relationship, especially for  $\kappa$  values around 1.

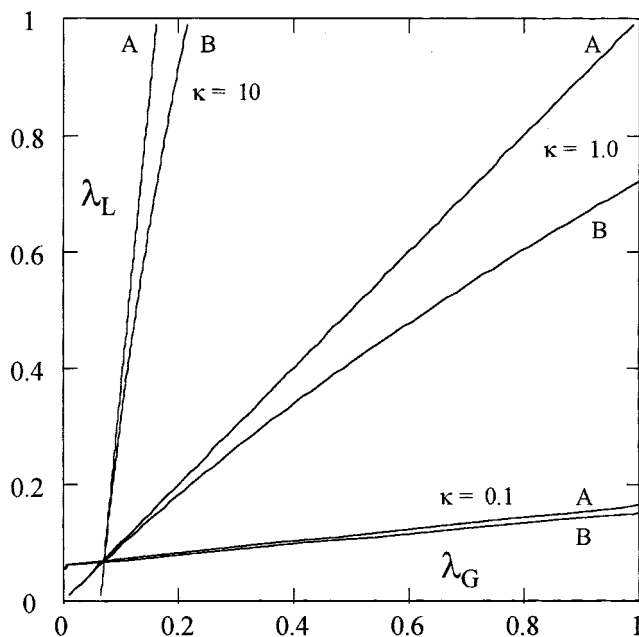


Figure 2. Gas-liquid flow split model lines in a regular dividing T junction.

Line A corresponds to Eq. 11 and line B to Eq. 13 with the ratio  $\beta_{L3}/\beta_{L1} = 2$ .

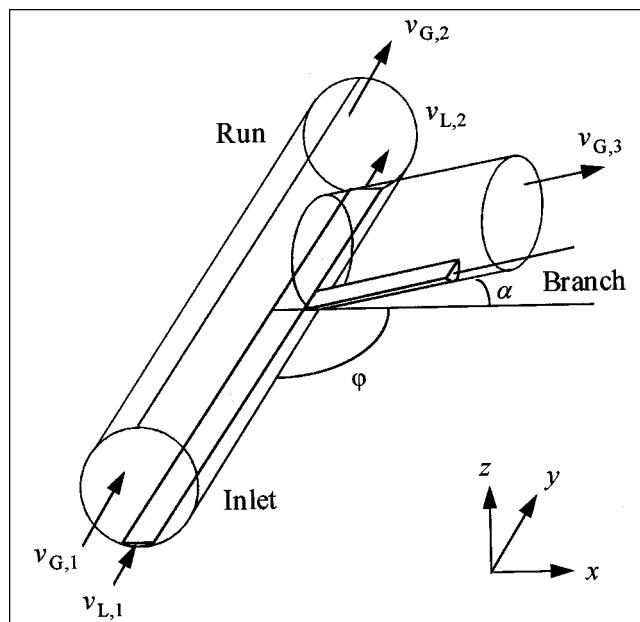


Figure 3. Geometry of a regular dividing T junction with liquid accumulation in the inclined branch.

#### Value of $\beta_L$ and $\lambda_{G,crit}$

In gas-transportation networks, pipes and junctions generally do not lie in a horizontal plane. In practice, most angles of inclination vary,  $-2^\circ < \alpha < 2^\circ$ . In a T junction with an upwardly laying branch (see Figure 3) the liquid starts to leave the branch at a critical gas mass intake fraction  $\lambda_{G,crit}$ . Consider a pressure-driven gas-sheared liquid layer (in the branch) with an interfacial shear stress  $\tau_i = f_i(1/2)\rho_G \langle v_G \rangle^2$ . In the two-dimensional case of a laminar flowing liquid layer the velocity distribution becomes

$$v_L = Y \frac{h_L^2}{2\eta_L} \left( 1 - \left( \frac{z}{h_L} \right)^2 \right) + \tau_i \frac{h_L}{\eta_L} \left( 1 - \frac{z}{h_L} \right), \quad (14)$$

in which:

$$Y = \left( \frac{dP}{dx} \right)_{TP} - \rho_L g \sin(\alpha). \quad (15)$$

The cross-sectional average liquid velocity becomes

$$\langle v_L \rangle = \frac{1}{h_L} \int_0^{h_L} v_L dz = Y \frac{h_L^2}{3\eta_L} + \tau_i \frac{h_L}{2\eta_L}. \quad (16)$$

The value of the velocity profile factor  $\beta_L$  of the liquid phase can be calculated using Eqs. 14–16:

$$\beta_L = \frac{\langle v_L^3 \rangle}{\langle v_L \rangle^3} = \frac{27}{35} \frac{70\tau_i^3 + 16Y^3h_L^3 + 77Y^2h_L^2\tau_i + 126Yh_L\tau_i^2}{(2Yh_L + 3\tau_i)^3}. \quad (17)$$

The values of  $Y$ ,  $h_L$ , and  $\tau_i$  are calculated with the holdup and pressure-gradient model of Hart et al. (1989) (see also Appendix A). The values of  $\beta_L$  for a laminar liquid film ( $Re_L = Re_{SL}/\theta < 2,000$ ) are now calculated using Eq. 17, while for a turbulent liquid film ( $Re_L \geq 2,000$ ), a value of  $\beta_L = 1.0$  is taken.

The critical-mass intake fraction  $\lambda_{G,crit}$  at which the liquid starts to leave the branch, can be derived from Eq. 16. If the average liquid velocity  $\langle v_L \rangle$  is negative, no liquid leaves the inclined branch of the regular T junction ( $\lambda_L = 0$ ), while for positive values of  $\langle v_L \rangle$  the liquid mass intake fraction can be obtained from Eqs. 10, 12, 13, and 17. The value of  $\lambda_{G,crit}$  is obtained from Eq. 16 when  $\langle v_L \rangle = 0$ .

Summarizing we can state that the  $\lambda_L - \lambda_G$  diagram for regular T junctions with a horizontal or slightly inclined branch can be obtained from

$$\lambda_L = \begin{cases} \frac{1}{2}(1 - \beta_{L,3}/\beta_{L,1})\lambda_L^2 + \lambda_0 + \kappa(\lambda_G - \lambda_0), & \text{if } \langle v_L \rangle \geq 0 \\ 0, & \text{if } \langle v_L \rangle < 0. \end{cases} \quad (18)$$

## Experimental Studies

### Gas-liquid flow loop

In Figure 4 an overview is given of the G/L pipe flow loop. The T junction is situated 8 m from the gas and liquid inlet. The inlet, run ( $L = 14$  m), and branch ( $L = 6$  m) consist of straight, horizontal glass tubes with an internal diameter of 50.8 mm. The glass T junction is radiused ( $r = R/D_1 = 0.00508$ ). The branch angle can be varied between  $-10^\circ$  and  $+10^\circ$ , while the horizontal inlet-to-run-tube deviates less than  $0.01^\circ$  from the horizontal. The pressure distribution around the T junction is measured with nine Validyne pressure transducers ( $\pm 5\%$ ). Water-saturated air ( $> 90\%$ ) compressed by a liquid ring pump ( $200 \text{ m}^3 \cdot \text{h}^{-1}$ ; Verder Vleuten) enters the flow loop through a turbine meter (Instromet  $\varnothing 75$  mm,  $\pm 1\%$ ) to determine the gas flow rate. The liquid flow is obtained from a storage vessel on an electronic on-line mass balance (Mettler-Toledo PM-16 & 30 K,  $\pm 0.1$ – $0.5$  g), placed eight meters above the injection point, that registers the

(constant) flow rate. Gas-liquid separators are placed at the end of the branch and run. The liquid flow rate is registered on-line at both ends, while the gas flow rate is determined at the outlet of the branch. Temperatures are registered at six points for accurately calculating the values of the transport properties of the fluids. All electronic sample devices are connected to a 486 microcomputer via a signal processing unit (Cambridge Electronic Design 1401). Within one experiment the difference between the six temperatures is held within 1 K. The superficial air velocity ranges between 5 and  $14 \text{ m} \cdot \text{s}^{-1}$ , and the superficial liquid velocity ranges between  $9 \times 10^{-4}$  and  $0.014 \text{ m} \cdot \text{s}^{-1}$ . The experiments are performed under atmospheric conditions ( $T = 293 \text{ K}$ ,  $P = 10^5 \text{ Pa}$ ). The transport properties of the gas-liquid systems used are  $\rho_G = 1.18 \text{ kg} \cdot \text{m}^{-3}$ ,  $\eta_G = 1.8 \times 10^{-5} \text{ Pa} \cdot \text{s}$ ; demineralized water:  $\rho_L = 998 \text{ kg} \cdot \text{m}^{-3}$ ,  $\eta_L = 1.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$ ; demineralized water/glycerol (33 wt. %):  $\rho_L = 1,080 \text{ kg} \cdot \text{m}^{-3}$ ,  $\eta_L = 2.7 \times 10^{-3} \text{ Pa} \cdot \text{s}$ ; demineralized water/glycerol (60 wt. %):  $\rho_L = 1,125 \text{ kg} \cdot \text{m}^{-3}$ ,  $\eta_L = 6.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$ .

### Measuring procedures

Several hundreds of steady-state experiments were carried out. All experiments started after the three glass arms were wetted by the liquid to prevent a hysteresis effect. After an angle of inclination  $\alpha$  is chosen and a value for  $\lambda_G$  is set by adjusting the valves at the end of the run and branch pipe, then a liquid flow is chosen and the system is allowed to obtain a steady state. Under steady-state conditions the pressure distribution around the T junction, the phase velocities, and other relevant physical quantities are measured. This procedure is repeated until the full range of  $\lambda_G$  is covered ( $0 < \lambda_G < 1$ ). The values of the friction loss coefficient  $k_{1,i,G}$  are calculated by extrapolating the measured pressure profile to the center of the T junction. The positions of the pressure taps are indicated in Figure 4, and the exact position can be extracted from Figure 5. The relations for calculating the experimental values of the friction-loss coefficients  $k_{12,G}$  and  $k_{13,G}$  during two-phase gas flow are

$$k_{1,i,G} = \frac{\langle v \rangle_{j,1}^2 - \langle v \rangle_{j,i}^2 + 2((P_1 - P_i)/\rho)_j}{\langle v \rangle_{j,1}^2} \quad i = 2, 3. \quad (19a,b)$$

## Results

### Pressure drop and friction-loss coefficients

In Figure 5 the experimentally determined gas-phase pressure distribution around the T junction is shown for a specific G/L flow split ( $\lambda_G = 0.83$ ,  $\lambda_L = 0.94$ ) at a branch inclination angle of  $0.25^\circ$ . As expected from the Bernoulli equations, it shows a pressure recovery in the run and a pressure drop in the branch as a result of pressure and velocity changes. The extrapolated pressures of the arms to the center of the T junction are depicted as  $P_1$ ,  $P_2$ , and  $P_3$ . From these values the junction irreversible pressure drops  $\Delta P_{12}$  and  $\Delta P_{13}$  are calculated and consecutively the values of the friction-loss coefficients  $k_{12,G}$  and  $k_{13,G}$  according to Eqs. 19a and 19b. Figure 6 shows that the correlation for single-phase flow developed by Gardel, that is, Eqs. 12a and 12b, can be used to calculate the difference in friction-loss coefficients

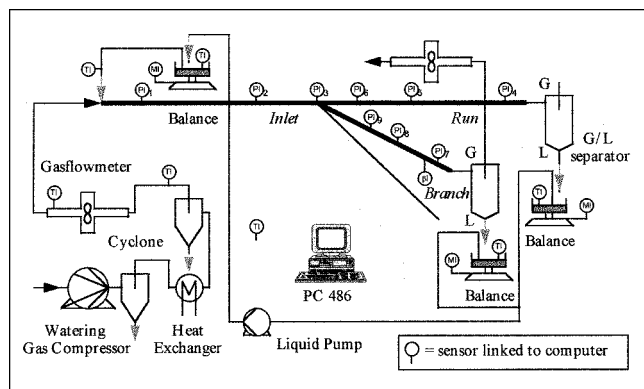


Figure 4. Experimental setup for measuring G/L flow split in T junctions with an inclined branch.

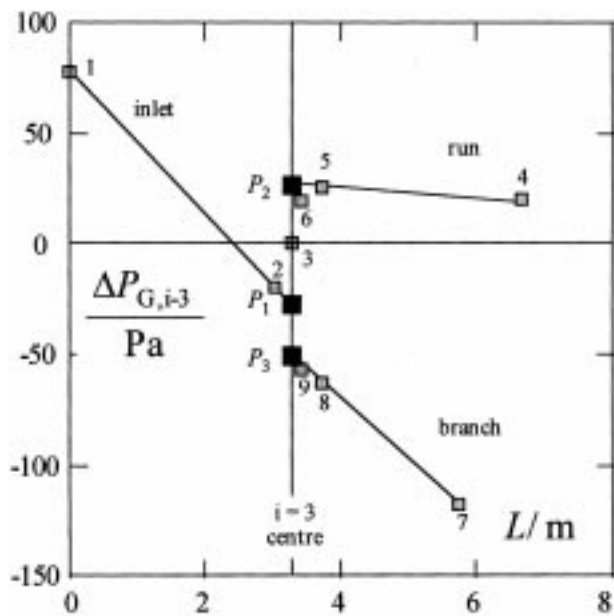


Figure 5. Experimental gas-phase pressure distribution around and in a T junction.

Typical gas-phase pressure distribution during G/L flow through a regular T junction with an inclined branch at  $\lambda_G = 0.83$  and  $\lambda_L = 0.94$ .  $v_{SG} = 10 \text{ m} \cdot \text{s}^{-1}$ ;  $v_{SL} = 4.0 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ ;  $\alpha = 0.25^\circ$ .

( $k_{13} - k_{12}$ ) occurring during two-phase G/L flow in horizontal T junctions with different branch inclinations. In the inclined case, however, at values of  $0.4 < \lambda_G < 0.6$ , a deviation is observed between the experimental two-phase values and

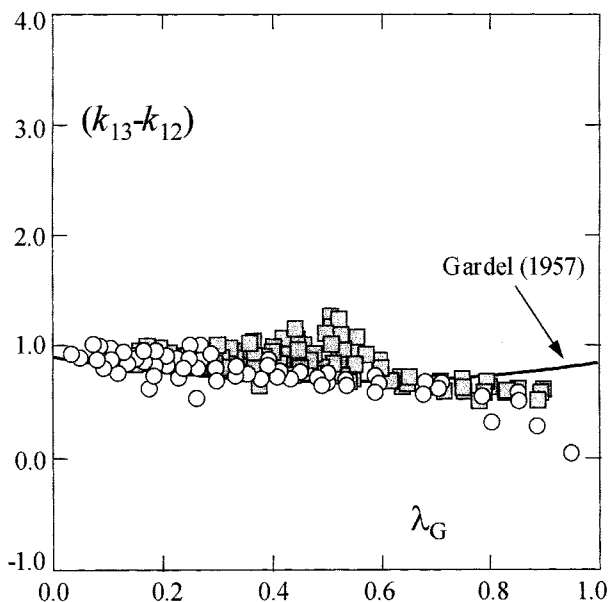


Figure 6. Friction loss coefficient difference ( $k_{13} - k_{12}$ ) as function of  $\lambda_G$ .

Air/water flow in a regular T junction with a horizontal (○) and an inclined branch (■) ( $\alpha = 0.1, 0.25, 0.5^\circ$ ). Line drawn according to Gardel (1957) for single-phase gas flow, that is, Eqs. 12a, 12b.

the model line of Gardel for single-phase flow. This deviation corresponds to the  $\lambda_G$  range, where the liquid starts to flow out of the branch arm, where disturbances in the pressure distribution occur.

### Gas-liquid flow split

In Figure 7 the values of  $\lambda_L$  and  $\lambda_G$ , calculated with the model (Eqs. 13–18), are compared with experimental data from Hart et al. (1991) concerning G/L flow split in a horizontal regular T junction. In the horizontal case a reasonable agreement is found between the lines calculated with the model and the experimental data from the literature. A remarkable jump is generated with the model in line E, which shows a resemblance to Azzopardi's "film-stop," but is, in fact, the transition from laminar to turbulent liquid-film flow. The effect of branch inclination on  $\lambda_L$  is clearly demonstrated in Figures 8–11.

Experimentally, it is observed that at the start of the liquid outflow from the inclined branch, the liquid layer is not distributed equally over the length of the branch. Nearby the T junction the liquid level is higher than at the end of the branch arm (at  $L = L_B$ ). This means that when we have to use the liquid height  $h_L$  for the calculation of the velocity distribution, we have to use a somewhat higher liquid level at the moment the liquid outflow of the branch starts, that is, at  $\lambda_{G,\text{crit}}$ . This difference in liquid height disappears when the liquid flow continuously leaves the branch. Although from geometrical considerations ( $h_{L,\text{end}} = 0$ , liquid volume is constant) a value of  $h_{L,\text{start}}$  of two times the calculated steady-state value for  $h_L$  is expected, experimental observations showed a value of 1.5 times the steady-state value  $h_L$ . After

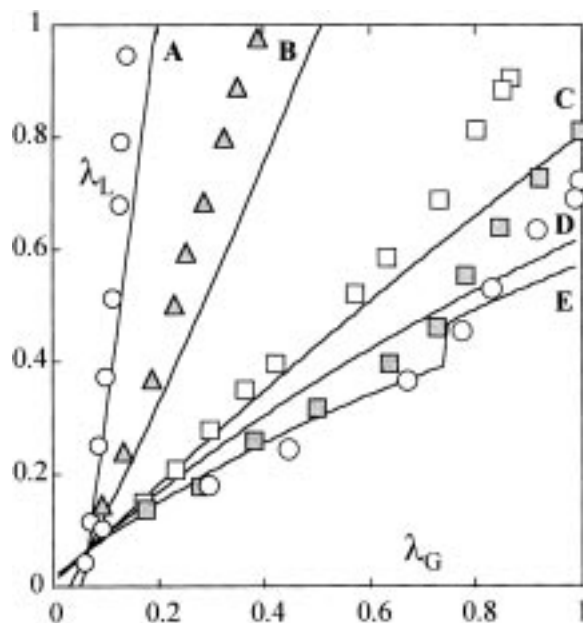


Figure 7. Model comparison with literature data of a regular horizontal T junction.

●, A:  $v_{SL} = 2.0 \times 10^{-4} \text{ m} \cdot \text{s}^{-1}$ ; ▲, B:  $v_{SL} = 1.57 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ ; □, C:  $v_{SL} = 7.21 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ ; ■, D:  $v_{SL} = 1.89 \times 10^{-2} \text{ m} \cdot \text{s}^{-1}$ ; ○, E:  $v_{SL} = 3.14 \times 10^{-2} \text{ m} \cdot \text{s}^{-1}$ . Lines (A–E) drawn according to Eqs. 13–18, with  $v_{SG} = 12 \text{ m} \cdot \text{s}^{-1}$ . Experiments according to Hart et al. (1991).

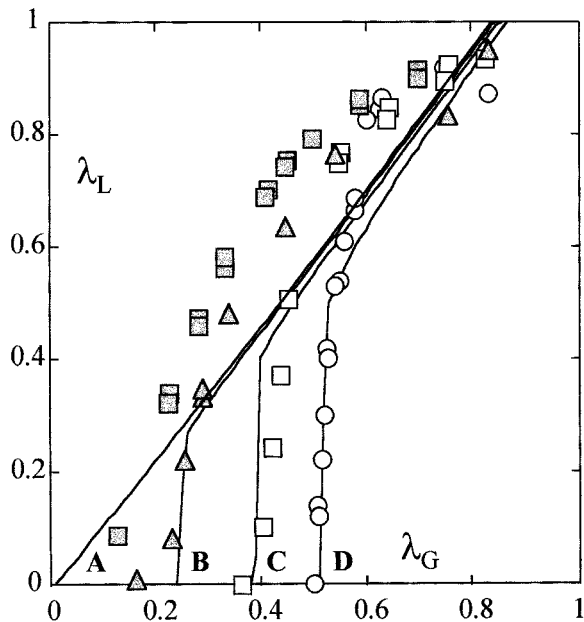


Figure 8. Effect of the branch inclination angle  $\alpha$ .

$\lambda_L$  as a function of  $\lambda_G$  for air/water flow in a regular T junction.  $v_{SG} = 10 \text{ m} \cdot \text{s}^{-1}$ ;  $v_{SL} = 4.0 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ ; ■, A:  $\alpha = 0.00^\circ$ ; ▲, B:  $\alpha = 0.10^\circ$ ; □, C:  $\alpha = 0.25^\circ$ ; ●, D:  $\alpha = 0.50^\circ$ . Lines drawn according to Eqs. 13–18.

the initial liquid outflow, the gradient disappears and the factor is set to 1 and thereby the steady-state value for  $h_L$  is used.

Figure 8 gives a good prediction of the critical  $\lambda_G$  value where the liquid starts to leave the branch ( $\lambda_{G,\text{crit}}$ ) for differ-

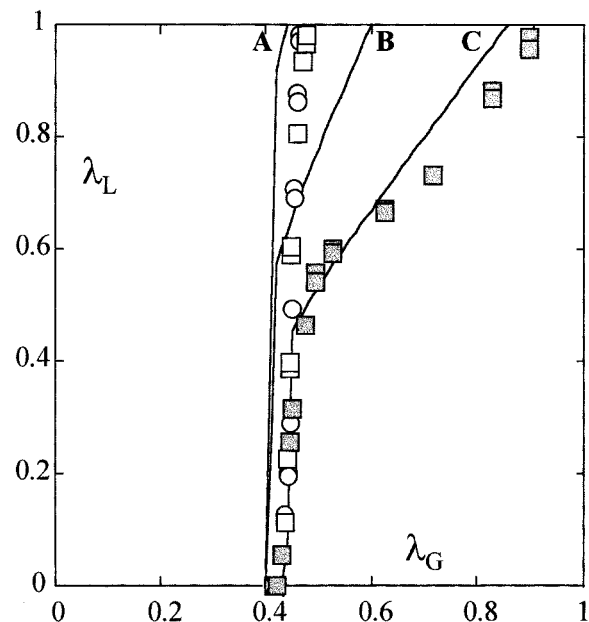


Figure 10. Effect of inlet superficial liquid velocity  $v_{SL}$ .

$\lambda_L$  as a function of  $\lambda_G$  for air/water flow in a regular T junction with a branch inclination of  $\alpha = +0.5^\circ$ .  $v_{SG} = 12 \text{ m} \cdot \text{s}^{-1}$ ; ●, A:  $v_{SL} = 9 \times 10^{-4} \text{ m} \cdot \text{s}^{-1}$ ; □, B:  $v_{SL} = 2 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ ; ■, C:  $v_{SL} = 4 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ . Lines drawn according to Eqs. 13–18.

ent angles of inclination. In the case of a horizontal T junction, the liquid enters the branch at  $\lambda_G \approx 0.1$ , while for an angle of  $\alpha = +0.5^\circ$  until  $\lambda_G = 0.5$  no liquid is observed at the branch outlet. The model lines in Figures 8–11 are drawn

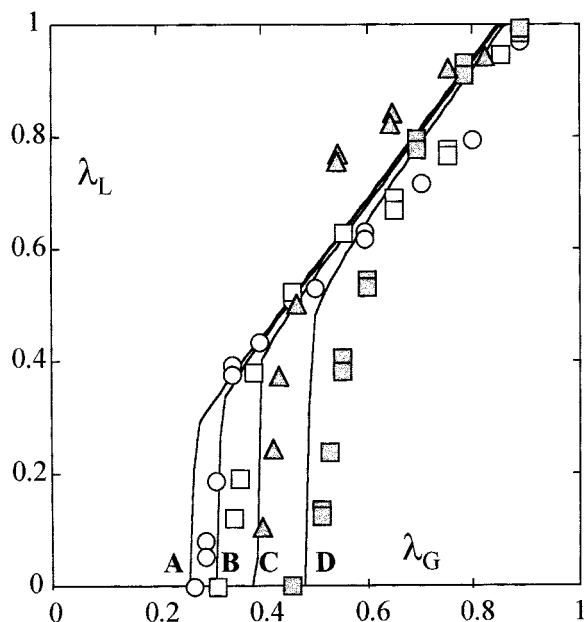


Figure 9. Effect of inlet superficial gas velocity  $v_{SG}$ .

$\lambda_L$  as a function of  $\lambda_G$  for air/water flow in a regular T junction. Branch inclination of  $\alpha = +0.25^\circ$ ;  $v_{SL} = 4 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ . ●, A:  $v_{SG} = 14 \text{ m} \cdot \text{s}^{-1}$ ; □, B:  $v_{SG} = 12 \text{ m} \cdot \text{s}^{-1}$ ; ▲, C:  $v_{SG} = 10 \text{ m} \cdot \text{s}^{-1}$ ; ■, D:  $v_{SG} = 8 \text{ m} \cdot \text{s}^{-1}$ . Lines drawn according to Eqs. 13–18.

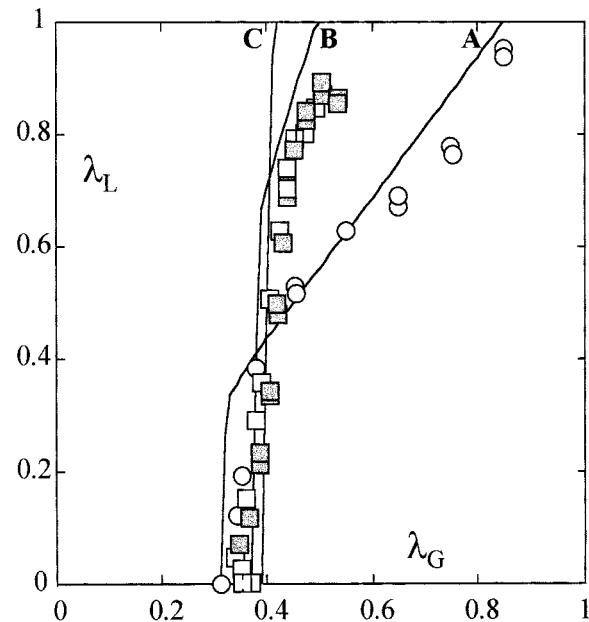


Figure 11. Effect of liquid viscosity  $\eta_L$ .

$\lambda_L$  as a function of  $\lambda_G$  for air/water and air/water–glycerol flow in a regular T junction with a branch inclination of  $\alpha = +0.25^\circ$ .  $v_{SG} = 12 \text{ m} \cdot \text{s}^{-1}$ ,  $v_{SL} = 4 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ . ●, A:  $\eta_L = 1 \text{ mPa} \cdot \text{s}$ ; □, B:  $\eta_L = 3 \text{ mPa} \cdot \text{s}$ ; ■, C:  $\eta_L = 6 \text{ mPa} \cdot \text{s}$ . Lines drawn according to Eqs. 13–18.

according to Eqs. 13–18, taking into account the just mentioned liquid buildup and liquid-layer gradient in the branch at the start of the liquid outflow (at  $\lambda_G = \lambda_{G,\text{crit}}$ ).

Figures 9 and 10 show the influence of the superficial velocity of the gas and liquid phase, respectively, in the inlet, and Figure 11 shows the effect of the dynamic liquid viscosity on the G/L flow separation in the T junction. The viscosity of the water–glycerol mixture is a factor of 3 to 6 higher than water, and a big difference is observed in the G/L splitting at the T junction. The increased viscosity of the liquid phase causes the splitting curve to shift to the right and to postpone the sharp decrease in the curve, at higher  $\lambda_G$  and  $\lambda_L$  values present in the air/water and air/water–glycerol experiment. Figures 7–11 show that the model presented in this section gives a good prediction of G/L separation in a regular T junction with a small inclination for the branch.

## Discussion

The pressure distribution shown in Figure 5 gives a good insight into the phenomenon responsible for the flow split. When distributing the gas phase over the two arms of the T junction, we see a pressure recovery in the run and a pressure drop in the branch, at a certain  $\lambda_G$ . The pressure barrier to overcome by the liquid film upon entering the run might be too high compared to its momentum. The result is a complete withdrawal of the liquid film in the branch. Using the relations of Gardel (1957) developed for single-phase flow for the case of two-phase flow proves fruitful. The value of  $\lambda_{G,\text{crit}}$  can be accurately predicted by the theory even though Figure 6 shows a deviation between the experimental and model values of the friction-loss coefficients.

Experimental values for the gradient of the  $\lambda_L$ – $\lambda_G$  curve strongly depend on  $\kappa$ , as was predicted by the theory. When we introduce a small branch inclination angle, for example,  $\alpha > 0.1$ , this leads to a higher value of the critical value of  $\lambda_G$ ,  $\lambda_{G,\text{crit}}$  and a sharp bend in the  $\lambda_L$ – $\lambda_G$  curve compared to the horizontal case. When we raise the value of  $\eta_L$ , this also leads to a higher value of  $\lambda_{G,\text{crit}}$  and a delay in the sharp bend in the  $\lambda_L$ – $\lambda_G$  curve. The model we developed follows the experiments on these three experimentally observed items.

When the liquid leaves the branch two phenomena are observed if the gas flow rate in the branch is increased. Either the value of  $\lambda_L$  rises steeply to 1.0 (complete separation), or after an initial sharp increase, it tends to the value observed in a horizontal T junction. These phenomena can be explained by the fact that when the liquid starts to flow, the liquid film thickness is not constant over the length of the inclined branch. A small increase in the gas velocity in the inclined branch results in a new steady-state situation in which the liquid level is constant. The liquid level approaches its threshold value of the horizontal case. This transition is thought to be responsible for the initial sharp increase in  $\lambda_L$ .

The effect of small branch inclination angles is substantial at moderate to low superficial gas velocities, for example, at  $v_{SG} < 20 \text{ m} \cdot \text{s}^{-1}$ . Figure 9 shows that the effect becomes increasingly important (gravitation vs. shear) when reducing the superficial gas velocity at the inlet from 14, 12, 10 to  $8 \text{ m} \cdot \text{s}^{-1}$ .

From Eq. 11, Eq. 13, and Figure 2 it is clear that the value of  $\kappa$  is an important parameter that governs the position and

gradient of the  $\lambda_G$ – $\lambda_L$  model line. From Figure 9 it can be seen that the value of the superficial gas velocity doesn't seem to have any influence on the position of the  $\lambda_G$ – $\lambda_L$  line, except in a shift in the value of the critical  $\lambda_G$ . The gradient of the  $\lambda_G$ – $\lambda_L$  model line is almost constant, and this can be explained by introducing the liquid holdup relation of Hart et al. (1989) for small values of the liquid holdup into Eq. 6 (see Appendix B), which leads to

$$\kappa \approx \frac{\beta_{G,1}}{\beta_{L,1}} 108 Re_{SL}^{-0.726}. \quad (20)$$

From Eq. 20 we can see that the value of  $\kappa$  is mainly a function of the superficial liquid velocity, or what we actually observe in practice. Hong (1978) also studied the effect of liquid viscosity on the flow split. He found that in a horizontal T junction an increased liquid viscosity results in an increased  $\lambda_L$  at constant  $\lambda_G$ . This is in accordance with our results in the inclined case, where after an initial delay  $\lambda_L$  rises to higher values at increased viscosities.

When this model is compared with data recently published in the work of Panmarcha et al. (1996), we see a remarkable difference between the data of Panmarcha et al. and the model. Our model predicts too high a value for the point where the liquid starts to leave the branch at increasing  $\lambda_G$ , the so-called critical  $\lambda_{G,\text{crit}}$  value. This is probably due to an effect we noticed when increasing  $\lambda_G$ . At a certain  $\lambda_G$ , a liquid-layer thickness gradient is present in a film that slowly creeps to the exit of the branch. In this situation, the value of  $\lambda_{G,\text{crit}}$  is dependent on the length of the branch. The conclusion could be that Panmarcha et al. (1996) did not take a branch long enough to let the liquid-layer thickness gradient disappear. It is advisable to take a minimum branch length of  $L_B \geq h_L/\sin(\alpha)$ ; otherwise, the liquid leaves the branch as a result of the hydrostatic pressure differences.

## Conclusions

Small branch inclination angles ( $\alpha \leq 0.5^\circ$ ) strongly affect gas–liquid flow separation in a regular T junction. Whereas the liquid phase may start to flow into the branch in the case of a horizontal T junction, in the inclined case, under the same operating conditions, only a layer of liquid is formed to a certain length, and no liquid flow is observed at the outlet of the branch. By increasing  $\lambda_G$ , the liquid eventually leaves the branch and a jump to full liquid separation is found ( $\lambda_L = 1$ ) or the horizontal split curve is approached after an initial steep rise.

The increase in viscosity of the liquid phase results in a shift of the splitting curve to the right, and to the disappearance or delay of the sharp decrease on the splitting curve, just after  $\lambda_{G,\text{crit}}$ .

To describe the G/L flow split behavior in a regular inclined T junction, the model presented in this article, made up of Eqs. 13–18, which are derived from the steady-state macroscopic energy balances of four streamlines, can be successfully used. Good agreement is found between the values calculated with the model and those obtained from experiments. Our experiments were performed with a constant branch length of 6 meters, and therefore we could not verify the effect of a longer branch arm on the G/L separation.



Summarizing, it can be concluded that the model—Eqs. 13–18—gives a good prediction of  $\lambda_L$  as a function of  $\lambda_G$  in the full range  $0 \leq \lambda_G \leq 1$ . Especially for moderate superficial gas velocities, the effect of small branch inclination angles is strong, and under these circumstances the model predicts well.

## Acknowledgments

The authors thank Mrs. J. Zoutberg, D. P. de Zwarte, Th. J. A. M. Nass, and W. H. Buster of the mechanical workshop for the construction of the gas–liquid flow loop, and for their technical support during the experiments.

## Notation

$D_i$  = average internal pipe diameter, m  
 $f$  = friction factor  
 $Fr$  = Froude number  
 $g$  = acceleration due to gravity,  $9.81 \text{ m} \cdot \text{s}^{-2}$   
 $h$  = height, m  
 $k_s$  = interfacial roughness, m  
 $L$  = pipe length, m  
 $L_x$  = length of branch filled with liquid, m  
 $P$  = pressure, Pa  
 $Re_G$  = Reynolds number of the gas phase  
 $Re_L$  = Reynolds number of the liquid film,  $Re_{SL}/\theta$   
 $Re_{SL}$  = superficial Reynolds number of the liquid phase,  $\rho_L v_{SL} D_i / \eta_L$   
 $t$  = time, s  
 $T$  = temperature, K  
 $v$  = phase velocity,  $\text{m} \cdot \text{s}^{-1}$   
 $w$  = root-mean-square velocity,  $\text{m} \cdot \text{s}^{-1}$   
 $x, y, z$  = Cartesian coordinates, m  
 $\delta$  = film thickness, m  
 $\epsilon$  = fraction of cross-sectional area occupied by a phase, holdup  
 $\Phi$  = mass flow rate,  $\text{kg} \cdot \text{s}^{-1}$   
 $\eta$  = dynamic viscosity of the liquid phase,  $\text{Pa} \cdot \text{s}$   
 $\theta$  = fraction of the tube wall wetted by the liquid phase  
 $\rho$  = density of a fluid,  $\text{kg} \cdot \text{m}^{-3}$

## Subscripts

1, 2, 3 = inlet, run, and branch arm  
 $R$  = run  
 $s$  = superficial  
 $TP$  = two phase

## Literature Cited

- Azzopardi, B. J., and B. P. Whalley, "The Effect of Flow Pattern on Two Phase Flow in a T Junction," *Int. J. Multiphase Flow*, **8**, 491 (1982).  
 Azzopardi, B. J., A. Purvis, and A. H. Govan, "Annular Two-Phase Flow Split at an Impacting T," *Int. J. Multiphase Flow*, **14**, 605 (1987).  
 Azzopardi, B. J., "The Split of Annular-Mist Flows at Vertical and Horizontal Ts," *Int. Conf. on Offshore Mechanics and Artic Engng.*, The Hague, The Netherlands, p. 389 (1989).  
 Ballyk, J. D., and M. Shoukri, "On the Development of a Model for Predicting Phase Separation Phenomena in Dividing Two-Phase Flow," *Nucl. Eng. Des.*, **123**, 67 (1990).  
 Bird, R. B., W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley, New York (1960).  
 Eck, B., *Technische Strömungslehre*, Springer-Verlag, New York (1973).  
 Gardel, A., "Les Pertes de Charge dans les Écoulements au Travers de Branchement en Té," *Bull. Tech. Suisse Romande*, **9**, 122, and **10**, 143 (1957).  
 Grolman, E., "Gas-Liquid Flow with Low Liquid Loading in Slightly Inclined Pipes," PhD Thesis, Univ. of Amsterdam, Amsterdam, The Netherlands (1994).

- Hart, J., P. J. Hamersma, and J. M. H. Fortuin, "Correlations Predicting Frictional Pressure Drop and Liquid Hold-up during Horizontal Gas-Liquid Pipe Flow with a Small Liquid Hold-up," *Int. J. Multiphase Flow*, **15**, 947 (1989).  
 Hart, J., P. J. Hamersma, and J. M. H. Fortuin, "A Model for Predicting Liquid Route Preference during Gas-Liquid Flow through Horizontal Branched Pipelines," *Chem. Eng. Sci.*, **46**, 1609 (1991).  
 Hong, K. C., "Two-Phase Flow Splitting at a Pipe Tee," *J. Pet. Tech.*, **2**, 290 (1978).  
 Hwang, S. T., H. M. Soliman, and R. T. Lahey, "Phase Separation in Dividing Two-Phase Flow," *Int. J. Multiphase Flow*, **14**, 439 (1988).  
 Marti, S., and O. Shaham, "A Unified Model for Stratified-Wavy Two-Phase Flow Splitting at a Reduced T Junction with an Inclined Branch Arm," *Int. J. Multiphase Flow*, **23**, 725 (1997).  
 McNow, J. S., "Mechanics of Manifold Flow," *ASCE Trans.*, **119**, 1103 (1954).  
 Oranje, L., "Condensate Behavior in Gas Pipelines is Predictable," *Oil Gas J.*, **71**, 39 (1973).  
 Ottens, M., A. de Swart, H. C. J. Hoefsloot, and P. Hamersma, "Gas-Liquid Flow Splitting in Regular, Reduced and Impacting T Junctions," *Proc. Conf. on Multi Phase Flow Dynamics in Industrial Plants*, Ancona, Italy (1994).  
 Panmatcha, V. R., P. J. Ashton, and O. Shoham, "Two-Phase Stratified Flow Splitting at a T Junction with an Inclined Branch Arm," *Int. J. Multiphase Flow*, **22**, 1105 (1996).  
 Roberts, P. A., "Two-Phase Flow at T Junctions," PhD Thesis, Univ. of Nottingham, Nottingham, England (1994).  
 Roberts, P. A., B. J. Azzopardi, and S. Hibberd, "The Split of Horizontal Semi-Annular Flow at a Large Diameter T Junction," *Int. J. Multiphase Flow*, **21**, 455 (1995).  
 Roberts, P. A., B. J. Azzopardi, and S. Hibberd, "The Split of Horizontal Annular Flow at a T Junction," *Chem. Eng. Sci.*, **52**, 3441 (1997).  
 Saba, N., and R. T. Lahey, "The Analysis of Phase Separation Phenomena in Branching Conduits," *Int. J. Multiphase Flow*, **10**, 1 (1984).  
 Seeger, W., R. Reimann, and U. Muller, "Two-Phase Flow in a T Junction with a Horizontal Inlet, Part I: Phase Separation," *Int. J. Multiphase Flow*, **12**, 575 (1986).  
 Shoham, O., J. P. Brill, and Y. Taitel, "Two-Phase Flow Splitting in a Tee Junction—Experiment and Modelling," *Chem. Eng. Sci.*, **42**, 2667 (1987).  
 Shoham, O., S. Arirachakaran, and J. P. Brill, "Two-Phase Flow Splitting in a Horizontal Reduced Pipe Tee," *Chem. Eng. Sci.*, **44**, 2388 (1989).

## Appendix A: Liquid Holdup and Pressure-Gradient Model

Hart et al. (1989) based their model on the assumption that the liquid phase could be considered as an artificial roughness. Therefore, the model is based only on the gas-phase momentum equation and does not incorporate the liquid-phase momentum equation. They used the friction factor relation of Eck (1973) as the basis for their  $f_f$  relation. They correlated the *sand roughness* of the relation of Eck to their experimental data through a Froude number and a wetted wall fraction. Their approach is as follows:

$$Re_{SL} = \rho_L v_{SL} D_i / \eta_L \quad (\text{A1})$$

Initial value for the liquid holdup:

$$\epsilon_L = \left\{ 1 - \left[ \frac{v_{SL}}{v_{SG}} \left( 1 + 10.4 Re_{SL}^{-0.363} \sqrt{\frac{\rho_L}{\rho_G}} \right) \right] \right\}^{-1} \quad (\text{A2a})$$

$$\epsilon_G = 1 - \epsilon_L \quad (\text{A2b})$$

$$v_L = v_{SL}/\epsilon_L \quad (\text{A3a})$$

$$v_G = v_{SG}/\epsilon_G \quad (\text{A3b})$$

$$Fr_L = \frac{\rho_L}{(\rho_L - \rho_G)} \frac{v_L^2}{gD_i} \quad (\text{A4})$$

$$\theta = 0.52 \epsilon_L^{0.374} + 0.26 Fr_L^{0.58} \quad (\text{A5})$$

$$\delta = \frac{\epsilon_L D_i}{4\theta} \quad (\text{A6a})$$

$$k_\delta = 2.3\delta \quad (\text{A6b})$$

$$Re_G = \rho_G v_G D_i / \eta_G \quad (\text{A7})$$

$$f_i = 0.0625 \left\{ \log \left( \frac{15}{Re_G} + \frac{k_\delta}{3.715 D_i} \right) \right\}^{-2} \quad (\text{A8})$$

$$f_L = f_i 108 Re_{SL}^{-0.726} \quad (\text{A9})$$

$$Fr_\delta = \frac{\rho_L}{(\rho_L - \rho_G)} \frac{v_L^2}{g\delta} \quad (\text{A10})$$

$$\epsilon_L = \epsilon_G \frac{v_{SL}}{v_{SG}} \left( 1 + \sqrt{\epsilon_G 108 Re_{SL}^{-0.726} \frac{\rho_L}{\rho_G}} \sqrt{1 + \frac{2 \sin(\alpha_T)}{f_L Fr_\delta}} \right). \quad (\text{A11})$$

Iterate until a converged solution for the liquid holdup is reached. Then calculate the pressure gradient:

$$f_G = 0.07725 \left\{ \log \left( \frac{Re_G}{7} \right) \right\}^{-2} \quad (\text{A12})$$

$$f_{TP} = (1 - \theta) f_G + \theta f_i \quad (\text{A13})$$

$$-\left( \frac{dP}{dx} \right)_{TP} = 4 f_{TP} \frac{1}{2} \rho_G v_G^2 \frac{1}{\epsilon_G D_i} - 4\theta f_i \frac{1}{2} \rho_G (2v_G v_L - v_L^2) \frac{1}{\epsilon_G D_i} + \rho_G g \sin(\alpha_T). \quad (\text{A14})$$

Subsequently the values of  $Y = (dP/dx)_{TP} - \rho_L g \sin(\alpha)$ ,  $h_L = \delta$ , and  $\tau_i = f_i(1/2)\rho_G v_G^2$  are obtained.

## Appendix B: $\kappa$ Relation

By definition (Eq. 6) it follows:

$$\kappa = \frac{\beta_{G,1}}{\beta_{L,1}} \frac{\rho_{G,1}}{\rho_{L,1}} \frac{\langle v_{G,1} \rangle^2}{\langle v_{L,1} \rangle^2} = \frac{\beta_{G,1}}{\beta_{L,1}} \frac{\rho_{G,1}}{\rho_{L,1}} \frac{v_{SG}^2}{v_{SL}^2} \left( \frac{\epsilon_L}{1 - \epsilon_L} \right)^2 \quad (\text{B1})$$

From Hart et al. (1989) for small liquid holdups ( $< 0.06$ ):

$$\frac{\epsilon_L}{1 - \epsilon_L} = \frac{v_{SL}}{v_{SG}} \left\{ 1 + \sqrt{(1 - \epsilon_L) 108 Re_{SL}^{-0.726} \frac{\rho_L}{\rho_G}} \times \sqrt{1 + \frac{2 \sin(\alpha_{T,1})}{f_L Fr_\delta}} \right\}, \quad (\text{B2})$$

assume a horizontal T junction, so  $\alpha_{T,1} = 0$ :

$$\left( \frac{\epsilon_L}{1 - \epsilon_L} \right)^2 = \left( \frac{v_{SL}}{v_{SG}} \right)^2 \left\{ 1 + \sqrt{(1 - \epsilon_L) 108 Re_{SL}^{-0.726} \frac{\rho_L}{\rho_G}} \right\}^2. \quad (\text{B3})$$

with low liquid holdup leads to approximately

$$\left( \frac{\epsilon_L}{1 - \epsilon_L} \right)^2 \cong \left( \frac{v_{SL}}{v_{SG}} \right)^2 108 Re_{SL}^{-0.726} \frac{\rho_L}{\rho_G}, \quad (\text{B4})$$

and for  $\kappa$ :

$$\kappa \cong \frac{\beta_{G,1}}{\beta_{L,1}} 108 Re_{SL}^{-0.726}. \quad (\text{B5})$$

Manuscript received July 31, 1995, and revision received Dec. 10, 1998.